Exercises 5

Exercise 1. (Rouché Theorem) Let $\mathcal{U}$ be an open set in $\mathbb{C}$, and let $D = \overline{B(a,r)}$ be any disc contained in $\mathcal{U}$. Let f, g be two meromorphic functions on $\mathcal{U}$. If

$$|f(z) - g(z)| < |g(z)|, \quad \forall z \in \partial D\,,$$

then

$$Z_f - P_f = Z_g - P_g\,,$$

where Z_f and Z_g are the number of zeros of f and g, respectively, and P_f and P_g are the number of poles of f and g, respectively.

Exercise 2. Compute the following integrals along the path $c : [0,1] \to \mathbb{C}$. Use the residue Theorem.

$$\int_c \frac{1}{z^4 + x^3 - 2z^2} dz, \qquad c(t) = 3e^{2\pi i t},$$
$$\int_c \frac{1}{z^4 + 4} dz, \qquad c(t) = 2e^{2\pi i t} + 1,$$
$$\int_c \frac{e^z}{z^3 + z} dz, \qquad c(t) = 2e^{2\pi i t}\,.$$

Exercise 3. Compute the following real integrals.

$$\int_{-\infty}^{\infty} \frac{1}{(x^2+1)(x^2+4)} dx,$$
$$\int_0^{\infty} \frac{\sin(x)}{x} dx\,.$$